

Optimal Asset Allocation for Retirement Portfolios

With the demise of traditional pensions and the rise of 401(k) plans, the responsibility of saving for retirement has shifted from the [employer to the employee](#). Many employees failed, and continue to fail, to take full advantage of their 401(k) plans, leaving them with insufficient funds for a comfortable retirement.

There are countless prescriptions for how to survive an under-funded retirement, but most of these answers were not obtained by rigorous analysis. Virginia Young, a mathematician, does provide a rigorous [answer](#) in a simplified version of reality.

Using such simplifications is common in the hard sciences. The goal is to retain the salient features of reality while omitting the less important details to facilitate calculations. Obviously, this approach has limitations because the mathematical framework describes only part of reality, not all of it.

Young's paper assumes that retirees can choose between two investment options:

1. A risk-free bond that pays a guaranteed fixed amount every year
2. A volatile stock with an expected return greater than that of the risk-free bond

All returns are assumed to be corrected for inflation, meaning that each interest payment from the bond has the same buying power. Bonds with these exact properties do not exist. Assuming their existence is one of the simplifications Young makes.

Retirees are then characterized by their total assets, how much they spend, and their mortality rate. A successful retirement is defined as still having assets at the time of death. In this simplified world, it is possible to calculate the allocation between the two assets that maximizes the probability of not running out of money.

These two examples illustrate the results:

The well-funded retiree:

This retiree can cover expenses by investing all assets in the risk-free bond and consuming the yield. This income stream continues forever because the principal remains untouched.

The under-funded retiree:

The under-funded retiree does not have enough assets to live on the yield of a risk-free bond portfolio. To cover expenses, he or she needs to consume some assets or invest more aggressively. Young showed that this person would consume all assets if they lived long enough.

The optimal asset allocation drops from nearly 100 percent risk-free bond for the richest under-funded retirees to 100% stock for the poorest ones. This is not the answer most financial advisors would give or one that clients would accept. Nevertheless, it is rational given that a successful retirement was defined as still having assets at the time of death.

Investing everything in the stock gives under-funded retirees a small chance of an outsized positive return that makes their assets last until they die. However, it also increases the probability of an outsized negative return, which would cause them to run out of money even faster than if they invested everything into the risk-free bond.

So why do advisors rarely tell under-funded clients to invest in the most aggressive investment they can find?

The real world is more flexible than the simplified version used to derive these answers!

Young's model does not permit any changes in the behavior of the retiree. In the real world, retirees can return to the workforce or reduce their expenses. Such changes have predictable consequences and a much higher chance of success than hoping for outsized investment returns.

Young's model becomes useful after all options to increase income and decrease expenses have been exhausted. It determines the only remaining adjustable parameter, the asset allocation between the stock and the bond. However, there are still very few advisors who tell their poorest clients to take the most investment risk.

Why do financial advisors not recommend this strategy?

The model's allocation is designed to maximize the probability of having assets left at the time of death. The more volatile the investment, the greater the probability of a return high enough to accomplish this. If maximizing this probability is the only consideration, advisors are irrational if they recommend anything short of the most aggressive investment options.

Young's model treats all scenarios where the retiree runs out of money before death as equally bad. Retirees and their advisors do not necessarily agree. They may think it is better to have a number of comfortable retirement years with the near certainty that they will run out of money later, rather than having a high risk

of very few comfortable retirement years and a small chance of being financially comfortable until they die.

Pivot Point Advisors is currently working on models that are similar to Young's but define retirement success in ways that may be closer to actual investor behavior. This is a natural extension of Young's rigorous method for finding rational answers to real-world questions. Her answer is not a rule of thumb such as 'you should have a 60/40 split of stocks and bonds in your portfolio' or 'hold a percentage of 100 minus your age in bonds.' It is the result of an application of scientific principles and mathematical rigor, as all results in finance and economics should be!

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